Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Friday 15 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z + p\mathrm{i}}{\mathrm{i}z + 3} \qquad z \neq 3\mathrm{i} \quad p \in \mathbb{Z}$$

The point representing $\mathrm{i}\left(1 + \sqrt{3}\right)$ is invariant under T.

Determine the value of p.

(3)

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Question 1 continued

(Total 3 marks)

Q1

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2. (a) Show that, for $r > 0$

$$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)}$$

(2)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{n(an+b)}{c(n+1)(n+2)}$$

where a, b and c are integers to be determined.

(4)

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Question 2 continued

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Question 2 continued

Q2

(Total 6 marks)

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3. Use algebra to obtain the set of values of x for which

$$\left|x^2 + x - 2\right| < \frac{1}{2}(x + 5)$$

(7)

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Question 3 continued

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Question 3 continued

Q3

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4. (a) Show that the substitution $y^2 = \dfrac{1}{z}$ transforms the differential equation

$$\frac{dy}{dx} + 2y = 3xy^3 \qquad y \neq 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4z = -6x \qquad \text{(II)}$$

(3)

(b) Obtain the general solution of differential equation (II).

(5)

(c) Hence obtain the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(1)

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Question 4 continued

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Q4

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5. Given that

$$\left(2 - x^2\right)\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 = 3y$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{1}{\left(2 - x^2\right)}\left(2x\frac{d^2y}{dx^2}\left(1 - 5\frac{dy}{dx}\right) - 5\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx}\right)$$

(5)

Given also that $y = 3$ and $\frac{dy}{dx} = \frac{1}{4}$ at $x = 0$

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

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Question 5 continued

Q5

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6. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 6\cos x$$

(7)

(b) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$

(5)

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Question 6 continued

Q6

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7.

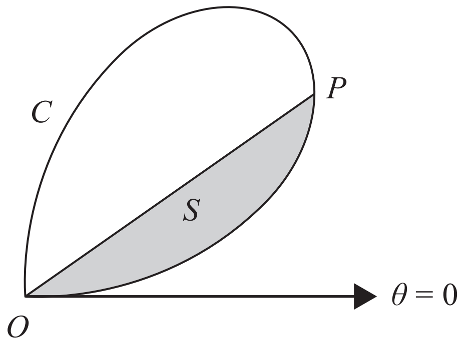

Figure 1

Figure 1 shows a sketch of curve C with polar equation

$$r = 3\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P on C has polar coordinates (R, ϕ). The tangent to C at P is perpendicular to the initial line.

(a) Show that $\tan\phi = \dfrac{1}{\sqrt{2}}$

(4)

(b) Determine the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and the line OP, where O is the pole.

(c) Use calculus to show that the exact area of S is

$$p\arctan\frac{1}{\sqrt{2}} + q\sqrt{2}$$

where p and q are constants to be determined.

Solutions relying entirely on calculator technology are not acceptable.

(7)

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Question 7 continued

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Question 7 continued

Question 7 continued

(Total 13 marks)

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Q7

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8. Given that $z = e^{i\theta}$

(a) show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0 \qquad 0 \leqslant \theta \leqslant \pi$$

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} \left(32\cos^6\theta - 4\cos^2\theta\right)d\theta$$

Solutions relying entirely on calculator technology are not acceptable.

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

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