OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y542/01: Statistics

Advanced GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$53.1 \pm 1.96\sqrt{\frac{30}{8}}$	**M1**	3.3	Correct structure with 8	
				A1	1.1	Square root correct	
				A1	1.1	Awrt 1.96 used, can be implied	
			(49.30, 56.90)	**A1**	3.4	Both, only these numbers (4 sf needed at least once)	Allow e.g. (49.30, 56.9)
				[4]			
2	(a)	(i)	The points do not lie very close to a straight line	**B1**	1.1	Or equivalent. Must refer to diagram, not just to "correlation"	Ignore extras unless wrong
				[1]			
		(ii)	H_0: $\rho = 0$, H_1: $\rho > 0$, where ρ is the population pmcc between prices in 1972 and prices in 2018	**B2**	1.1 2.5	One error, e.g. ρ not defined, B1 (but allow "population" not stated) H_0: $r = 0$, H_1: $r > 0$: same scheme, but B2 needs "population" pmcc	H_0: no correlation, H_1: positive correlation: B1
			$0.381 < 0.4973$	**M1**	1.1	Compare with 0.497(3)	
			Do not reject H_0.	**M1ft**	1.1	Correct first conclusion, needs like-with-like	FT on CV 0.5760 only
			There is insufficient evidence of (positive) correlation between prices in the two years.	**A1ft**	2.2b	In context, not too definite	
				[5]			
		Exx	α: Insufficient evidence to reject H_0. No correlation between … M1A1 (bod)				
			β: Wrong first conclusion, correct interpretation: M0A0				
			γ: Hypotheses wrong way round: maximum M1M1				
	(b)		0.650	**B2**	3.1a	Full marks for correct answer by any method	SC: if B0 allow B1 for any 3 of 8.85, 46.35, 8.8725, 241.7331, 43.153
				[2]	1.1		

Question			Answer	Marks	AO	Guidance	
3	(a)		H_0: $m_A = m_B$, H_1: $m_A < m_B$ where m_A and m_B are the median journey times for A and B	**B1**	1.1	*OR:* Median journey times equal, oe. Allow if *m*s used but not defined	Allow "mean" or "average" only if "population" stated
			$W \sim N(180, 510)$	**B1**	1.1	Both, can be implied, needs $m = 12$	Allow $\sqrt{510}$ or 510^2
			Consider correct tail, either 219 or 141 ($R_m = 219$, $m(m+n+1) - R_m = 141$)	**M1**	1.1	Find *either* P(≥ 219) (218.5) *or* P(≤ 141) (141.5)	Use of 0.9559 is M0 here. For CV method see below
			$p = \Phi\left(\frac{141.5-180}{\sqrt{510}}\right) = 0.0441\ldots$ **BC**	**M1** **A1**	1.1 1.1	Needs *some* evidence. E.g.: 0.0421, 0.0401, 0.470 (no/wrong cc, √): M1	
			$0.0441 < 0.1$	**A1ft**	1.1	Explicit comparison. FT on wrong *p*-value provided method correct	$0.9559 > 0.9$: A1A1 (M1A1) $0.9559 > 0.1$: A1A0 M0A0
		OR:	CV $180 - z \times \sqrt{510}$	**M1**		Allow √ errors	$180 + 1.282\sqrt{510}$ etc is M0
			141 (141.5) used	**M1**			*unless* 219 (218.5) used, in
			$z = 1.282$ (CV = 151.05, 151.058..)	**A1**		Stated or implied	which case give M2(A1A1)
			$141.5 < 151.05(85)$ or $218.5 > 208.95$	**A1**		CV and cc correct e.g. $141 < 150.55$	E.g. $219 > 209.45$
			Reject H_0.	**M1ft**	1.1	Correct first conclusion	Needs like-with-like, e.g.
			Significant evidence that route B takes longer	**A1ft** **[8]**	2.2b	Contextualised, not too definite	0.9559 with 0.9
			SC Sum of A's ranks = 435 – 219 = 216 used: B1B0 M0M1A0A1 M1A1 max 5/8				
		Exx	α: H_0: Journey times are the same, H_1: journey times for B are higher: B0 β: H_0: No evidence that median journey times are different, etc: B0				
	(b)		Must be a random sample (of all journeys)	**B1**	3.5b	Or equivalent.	*Not* "representative".
			Or distributions must be same shape (necessary assumption for Wilcoxon rank-sum test!)	**[1]**		Allow "(journeys) independent"	

Question			Answer	Marks	AO	Guidance	
4			$3E(X) = 30$ or $E(X) = 10$	**B1**	2.2a	Used, stated or implied	
			$9 \times \text{Var}(X) = 36$ or $\text{Var}(X) = 4$	**B1**	2.2a	One of these, used, stated or implied	
			$\frac{1}{12}(n^2 - 1) = 4$	**M1**	1.1	Use variance of uniform	
			$\Rightarrow n = 7$	**A1**	2.2a	$n = 7$ only, no need for "reject –7"	
			$E(X - m) = \frac{1}{2}(n + 1)$	**M1**	3.1b	Use expectation of uniform, e.g. $2m + n + 1 = 20$.	Allow if $E(3X + m)$ used rather than $E[3(X + m)]$
		OR:	$\text{Var}(Y + m) = \frac{1}{12}(n^2 - 1)$	**M1**	1.1		
			$\Rightarrow n = 7$	**A1**	2.2a	$n = 7$ only, no need for "reject –7"	
			$E(Y + m) = ½(n + 1) + m$	**M1**	3.1b	Use expectation of uniform, e.g. $2m + n + 1 = 20$.	
			$10 - m = 4$	**M1**	2.1	Validly derive single equation for m	
			$m = 6$	**A1**	2.2a	$m = 6$ only	NB: $\text{Var} = (n - 1)^2/12$ is from *continuous* uniform!
				[7]			

Question			Answer	Marks	AO	Guidance	
5	(a)		${}^5C_3 \times {}^{21}C_2 + {}^5C_4 \times {}^{21}C_1 + 1$ [= 2100 + 105 + 1]	M1dep A1	3.1b 1.1	Any correct pair of ${}^nC_r$s multiplied All terms correct	Or 1 – P(0, 1, 2) = 1 – .9665
			$\div {}^{26}C_5$ [= 65780]	*depM1	1.1		
			$\frac{1103}{32890}$ or 0.0335…	A1 [4]	3.2a	Awrt 0.0335 or any exact fraction	e.g. $\frac{2206}{65780}$ or $\frac{264720}{7893600}$
		OR:	*Or:* $\frac{5}{26}\times\frac{4}{25}\times\frac{3}{24}\times\frac{2}{23}\times\frac{1}{22}$	B1			
			$\frac{5}{26}\times\frac{4}{25}\times\frac{3}{24}\times\frac{2}{23}\times\frac{21}{22}\times 5$	B1		Must have 5 oe, e.g. ${}^5C_1$	
			$\frac{5}{26}\times\frac{4}{25}\times\frac{3}{24}\times\frac{21}{23}\times\frac{20}{22}\times 10$	B1		Must have 10 oe, e.g. ${}^5C_3$	
			Total $\frac{1103}{32890}$ or 0.0335…	B1 [4]			
	(b)	(i)	$\frac{22!\times 5!}{26!}$ $(=\frac{1\times2\times3\times4\times5}{23\times24\times25\times26}=\frac{120}{358800})$	M1	1.1	Oe. Allow M1 for 21! instead of 22!	$\frac{1\times2\times3\times4\times5}{22\times23\times24\times25\times26}$: M1
			$=\frac{1}{2990}$ **AG**	A1 A1 [3]	2.1 2.2a	Fully correct Correctly obtain AG using exact method	Allow even if no working after 22! × 5! ÷ 26!
		(ii)	22 fences: 22 for [VVV] × 21 for [VV]	M1	3.1b	Correct strategy, allow ${}^{22}C_2$ for ${}^{22}P_2$	21!×3!×2!×22×21: M2A0
			Consonants arranged in 21! ways	M1	1.1	At least one of these, no subtraction	21! × 3! × 2! ÷ 26! M0M1
			Vowels arranged in 5! ways (= ${}^5P_3 \times {}^2P_2$)	A1	2.1	Both correct	${}^5C_3 \times 3! \times 2! = 5!$
			Product ÷ 26! = $\frac{21}{2990}$ (= $2.832\times10^{24} \div 4.0329\times10^{26}$)	A1 [4]	3.2a	Allow from calculator but must be exact fraction	
		OR:	Treat 21 consonants, [VVV] and [VV] as 23	M1	3.1b	Correct strategy, allow 23!×2!×3!	(Must subtract 2 × 1/2990 as 23! method counts [VVVVV] twice, once as [VVV][VV] and once as [VV][VVV])
			23! × 5! / 26! (= 1/130)	A1	2.1	Correct (5! = ${}^5P_3\times{}^2P_2 = {}^5C_3\times2!\times3!$)	
			Subtract 2 × 1/2990	M1	3.2a	M1 also for subtracting 1 × 1/2990	
			Answer is $\frac{21}{2990}$	A1 [4]	1.1	Final answer, exact fraction (11/1495 is M1A1M1A0)	

Question			Answer	Marks	AO	Guidance
6	(a)		Any reason for independence (or not) … and for constant average rate (or not), in each case without misunderstanding of what they mean	**B1** **B1** **[2]**	3.5b 3.5b	"Events occur independently and at constant average rate": B0 SC: Mere assertion of both, properly contextualised: B1 SC: Variance = 4.67 which is closer to 5: B1 SC: Considers only the assumptions given in the question: B0
	(b)	(i)	0.146(223) **BC**	**M1** **A1** **[2]**	3.4 1.1	Correct method stated or implied Correct answer only, awrt 0.146
		(ii)	0.133(372) **BC**	**M1** **A1** **[2]**	1.1 1.1	0.068: M1A0 (treat 0.1337 as a slip, i.e. give A1 BOD)
	(c)		Po(12.2) $P(\leq 15) - P(\leq 9)$ [= 0.8296 – 0.2253] = 0.604(224) **BC**	**M1** **M1** **A1** **[3]**	3.3 1.1 3.4	Stated or implied Allow $P(\leq 16)$ or $P(\leq 10)$, e.g. 0.503 or 0.662 (M1M1A0) Allow this M1 also from $\lambda = 7.2$ (0.187, 0.110, 0.189) Correct answer only, awrt 0.604
	(d)		Sales of CD players and integrated systems need to be independent	**B1** **[1]**	1.1	Need "independent" or "not related" clearly referred to the two types of machine. *Not just* "purchases independent" or "distributions independent"
	(e)		If a customer buys a CD player they probably won't (or will) buy an integrated system as well	**B1** **[1]**	3.5b	Any reason for non-independence of sales of CD players and integrated sound systems Can get B0B1 provided they are focussing on independence
		Exx	α: May buy both so not independent: B0 β: Often bought together: B1 γ: Misunderstanding of context, e.g. CDs/CD players, or assuming that integrated systems don't include CD players: can get B1			

Question			Answer	Marks	AO	Guidance	
7	(a)		Geometric	M1	1.1	Stated explicitly	
			Mean = 400 ÷ 100 (= 4) and p = 1/mean	M1	2.4	Use mean (or P(1) etc) to deduce p ("Determine", so justification is needed for 0.25)	Needs to deduce p in part **(a)**, not defer it to **(b)**
			Therefore $p = 0.25$	A1 [3]	1.1	Allow even if second M1 not gained	SC Geo(0.2) using P(1) = 0.2: M1M1A0
	(b)		Probability is 0.75^6 (= 0.1779785…)	M1	3.3		SC Geo(0.2): 0.8^6 M1A0
		OR:	*Or:* 0.177978 or 0.177979 or better seen, *or* 1 – [P(1)+ … + P(6)] with evidence, e.g. formula	M1		Allow ± 1 term	
			Expected frequency = probability×100 = 17.798	A1 [2]	2.1	17.798 correctly obtained, with sufficient evidence, www	100 – Σ(other frequencies): SC B1
	(c)		H_0: data consistent with (geometric) distribution	B1	1.1	Both, allow equivalents, but not "evidence that …".	E.g. H_0: $X \sim$ Geo(p) Allow Geo(0.25)
			H_1: not consistent				
			$\Sigma X^2 = 9.005$	B1	1.1	9.005 or 9.01	
			$9.005 < 11.07$ ($\nu = 5$)	B1	1.1	Compare their ΣX^2 with 11.07	
			Do not reject H_0.	M1ft	1.1	Correct first conclusion, ft on their 9.005 or on 12.59, needs like-with-like	Allow from comparison with 12.59 but nothing else
			Insufficient evidence that a geometric distribution is not a good fit.	A1ft [5]	2.2b	Contextualised, not too definite (needs double negative) Don't penalise "Geo(0.25)"	Allow addition slip in ΣX^2 SC Geo(0.2): can get full marks if given data used, $\Sigma X^2 = 4.54$ used gets B1B1B0M1A1
		Exx	α: Reject H_0. Data is consistent with geometric: M1A0				
			β: Insufficient evidence to reject H_0. Data is consistent with geometric: M1A1 (BOD)				

Question			Answer	Marks	AO	Guidance	
8	(a)		$\int_1^\infty kx^{-n}\,dx = \left[\frac{k}{(1-n)x^{n-1}}\right]_1^\infty$	**M1**	1.1	Integral attempted, correct limits	Don't need full details of $\lim(a \to \infty)$
				B1	1.1	Correct indefinite integral	
			$= \frac{k}{n-1} = 1$ so $k = n-1$	**A1** [3]	1.1	Correctly obtain $k = n-1$, www	
	(b)	(i)	$\int 3x^{-4}\,dx = -\frac{1}{x^3} + c$	**M1**	1.1	Needs + c or definite integral between 1 and x, oe	Wrong k: can get M1A0B1
			$x = 1$, $F(x) = 0$ so $c = 1$. Hence $1 - x^{-3}$.	**A1**	1.1	Fully correct active part of CDF	Ignore ranges here
			$F(x) = \begin{cases} 0 & x < 1, \\ 1 - \frac{1}{x^3} & x \geq 1 \end{cases}$	**B1** [3]	1.1	"0 for $x < 1$" stated and no wrong ranges (doesn't need M1 or A1) Allow ≤ for <, and/or > for ≥	Or "0 otherwise" if "$x \geq 1$" stated in active part
		(ii)	$\frac{P[(X>7)\cap(X>5)]}{P(X>5)} = \frac{P(X>7)}{P(X>5)}$	**M1***	3.1a	Use conditional probability method	$\frac{[1-F(7)][1-F(5)]}{1-F(5)}$: M1A0M0A0
				A1	3.1a	$P[(X>7)\cap(X>5)] = P(X>7)$	
			$= \frac{1-F(7)}{1-F(5)}$	***depM1**	3.3	Convert probabilities into $F(X)$, *not* using $P(X>7) \times P(X>5)$	
			$= \frac{125}{343}$ or 0.364(431…)	**A1ft** [4]	1.1	Any exact fraction or awrt 0.364, ft on $1 - a/x^3$, $a \neq 0, 1$	Allow from $F(x) = 1 - a/x^3$, otherwise www
	(c)		$E(X^2) = \int_1^\infty kx^{2-n}\,dx = \left[\frac{kx^{3-n}}{(3-n)}\right]_1^\infty \ (n \neq 3)$	**M1***	2.1	Correct limits needed somewhere	SC: $E(X^2) = \frac{n-1}{n-3}$, M1B1
				B1	1.1	Correct indefinite integral or $\frac{n-1}{n-3}$	$E(X) = \frac{n-1}{n-2} \Rightarrow n \neq 2$ or 3: (not valid, must consider ln if $n = 2$ or 3): B0
			If $n = 3$, $E(X^2) = \lim_{x\to\infty}[2\ln(x)]$, not defined			No marks just for this unless last 3 marks all zero, then if this (or for $n = 2$) is shown, award SC B1	No limits used: M0B1M0B0
			Infinite integral does not converge if $3 - n \geq 0$	***depM1**	2.2a	Make deduction based on convergence, ft	
			If $n \geq 4$ then $E(X) = \left[\frac{kx^{2-n}}{(2-n)}\right]_1^\infty$ converges	**B1**	2.3	Consider convergence of $E(X)$	SC: $Var(X) < 0$ when $n < 3$: M1B1M1 (B0) A0
			Therefore $Var(X)$ is not defined if and only if $n = 2$ or 3.	**A1** [5]	2.2a	Shown not defined for $n = 2$ or 3 *and only* for those	But no need to state "if and only if"

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