OCR
Oxford Cambridge and RSA

Monday 3 June 2019 – Morning

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

The quadratic equation $x^2 - 2x + 5 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. **[1]**

(b) Hence find a quadratic equation with roots $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. **[3]**

2 Indicate by shading on an Argand diagram the region

$\{z : |z| \leqslant |z - 4|\} \cap \{z : |z - 3 - 2\mathrm{i}| \leqslant 2\}$. **[3]**

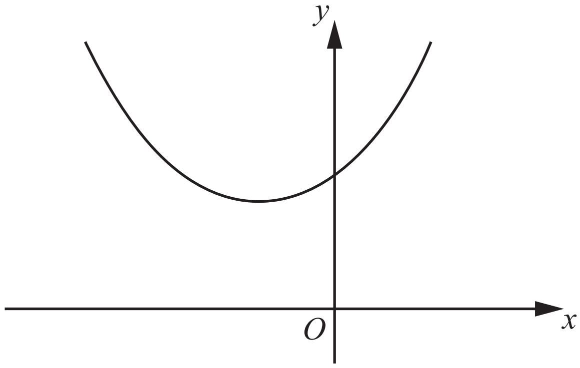
3 **In this question you must show detailed reasoning.**

You are given that $x = 2 + 5\mathrm{i}$ is a root of the equation $x^3 - 2x^2 + 21x + 58 = 0$.

Solve the equation. **[4]**

4 Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{10} r(3r - 2) = 1045$. **[3]**

5 The diagram shows part of the curve $y = 5\cosh x + 3\sinh x$.



(a) Solve the equation $5\cosh x + 3\sinh x = 4$ giving your solution in exact form. **[4]**

(b) **In this question you must show detailed reasoning.**

Find $\int_{-1}^{1}(5\cosh x + 3\sinh x)\,dx$ giving your answer in the form $ae + \frac{b}{e}$ where a and b are integers to be determined. **[3]**

6 You are given that $y = \tan^{-1}\sqrt{2x}$.

(a) Find $\frac{dy}{dx}$. **[2]**

(b) Show that $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)}\,dx = k\pi$ where k is a number to be determined in exact form. **[4]**

7 The function sech$x$ is defined by $\operatorname{sech} x = \frac{1}{\cosh x}$.

(a) Show that $\operatorname{sech} x = \frac{2e^x}{e^{2x}+1}$. **[2]**

(b) Using a suitable substitution, find $\int \operatorname{sech} x\,dx$. **[4]**

8 The equation of a plane is $4x+2y+z=7$.
The point A has coordinates (9, 6, 1) and the point B is the reflection of A in the plane.

Find the coordinates of the point B. **[6]**

9 **In this question you must show detailed reasoning.**

You are given the complex number $\omega = \cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$ and the equation $z^5 = 1$.

(a) Show that ω is a root of the equation. **[2]**

(b) Write down the other four roots of the equation. **[1]**

(c) Show that $\omega + \omega^2 + \omega^3 + \omega^4 = -1$. **[2]**

(d) Hence show that $\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$. **[3]**

(e) Hence determine the value of $\cos\frac{2}{5}\pi$ in the form $a + b\sqrt{c}$ where a, b and c are rational numbers to be found. **[4]**

10 You are given the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$.

(a) Find, in terms of a, the determinant of $\mathbf{A}$, simplifying your answer. **[2]**

(b) Hence find the values of a for which $\mathbf{A}$ is singular. **[2]**

You are given the following equations which are to be solved simultaneously.

$$\begin{aligned} ax + 2y \phantom{{}+ 2z} &= 6 \\ ay + 2z &= 8 \\ 4x + 5y + z &= 16 \end{aligned}$$

(c) For each of the values of a found in part **(b)** determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution. **[7]**

11 A particle is suspended in a resistive medium from one end of a light spring. The other end of the spring is attached to a point which is made to oscillate in a vertical line.

The displacement of the particle may be modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 10\sin t$$

where x is the displacement of the particle below the equilibrium position at time t.

When $t = 0$ the particle is stationary and its displacement is 2.

(a) Find the particular solution of the differential equation. **[11]**

(b) Write down an approximate equation for the displacement when t is large. **[2]**

END OF QUESTION PAPER

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