Oxford Cambridge and RSA

GCE

Further Mathematics A

Y540/01: Pure Core 1

Advanced GCE

Mark Scheme for June 2019

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

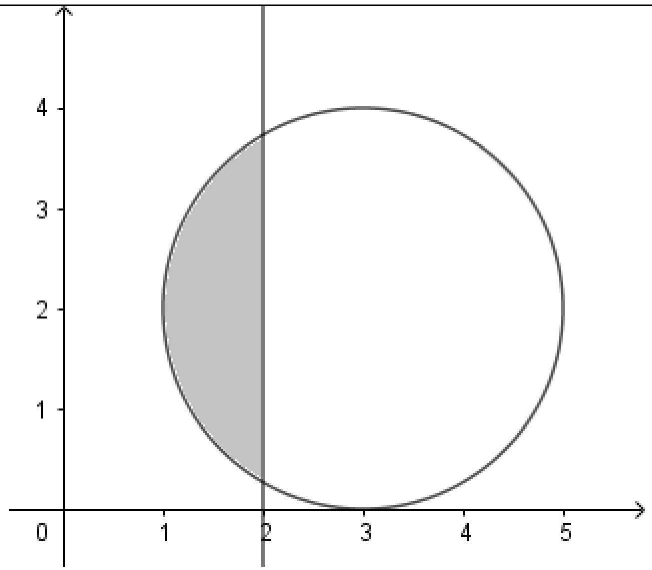
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		**DR** $\alpha+\beta=2,\ \alpha\beta=5$	**B1** **[1]**	1.1		
	(b)		**DR** $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=\alpha+\beta+\left(\frac{\alpha+\beta}{\alpha\beta}\right)=2+\frac{2}{5}=\frac{12}{5}$ $\left(\alpha+\frac{1}{\beta}\right)\times\left(\beta+\frac{1}{\alpha}\right)=\alpha\beta+2+\frac{1}{\alpha\beta}=7+\frac{1}{5}=\frac{36}{5}$ $\Rightarrow 5x^2-12x+36=0$ **oe**	**M1** **A1** **A1** **[3]**	1.1a 1.1 2.2a	Attempt both sum and product of new roots in terms of original roots For one of 12/5 or 36/5 For both, correctly interpreted as quadratic.	DR so finding roots M0

Question			Answer	Marks	AO	Guidance	
2				**B1** **B1** **B1** **[3]**	2.2a 2.2a 1.1	Correct circle Centre 3+2i radius 2 Correct line Re(z) = 2 Correct shading.	"correct" requires clear indication of centre and touching x-axis Alternatively, candidates may shade the region that **is not** required, but should indicate clearly that what they have shaded is not required.

Question			Answer	Marks	AO	Guidance	
3			**DR** Second root is the conjugate of $2+5\text{i}$, so $x=2-5\text{i}$ **soi**	**B1**	2.2a	Soi by correct quadratic $x^2-4x+29$	
			So the cubic $x^3-2x^2+21x+58=0$ can be written $\left(x-a\right)\left(x-(2+5\text{i})\right)\left(x-(2-5\text{i})\right)=0$	**M1**	1.1	Attempt to factorise using complex conjugate	Any valid method to find real root by reasoning, including division, or listing or using the factor theorem or sum of roots.
			$\Rightarrow -a(2+5\text{i})(2-5\text{i})=58$ $\Rightarrow a=-2$	**A1**	2.1	Shown convincingly **oe** (i.e. $(x+2)$ seen	NB. A DR question so full reasoning must be shown.
			So the solution of the cubic is $\Rightarrow x=-2,\ 2\pm5\text{i}$	**A1**	1.1		
				[4]			

Question			Answer	Marks	AO	Guidance	
4			$\sum_{r=1}^{10} r(3r-2) = \sum_{r=1}^{10}(3r^2-2r) = 3\sum_{r=1}^{10} r^2 - 2\sum_{r=1}^{10} r$	M1	1.1	Separate **soi**	
			$= 3\left(\frac{1}{6}10.11.21\right) - 2\left(\frac{1}{2}10.11\right)$	M1	2.1	Use both formulae with $n = 10$	
			$\left(= 55(21-2) = 55\times 19\right)$				**oe** 1155 – 110
			$= 1045$	A1	1.1	**AG**	
			Alternative Q3 leaving substitution to the end				
			$\sum_{r=1}^{10} r(3r-2) = \sum_{r=1}^{10}(3r^2-2r) = 3\sum_{r=1}^{10} r^2 - 2\sum_{r=1}^{10} r$ **M1**				
			$= \frac{3}{6}n(n+1)(2n+1) - \frac{2}{2}n(n+1)$				
			$= \frac{1}{2}n(n+1)(2n+1-2) = \frac{1}{2}n(n+1)(2n-1)$				
			$n = 10 \Rightarrow \frac{1}{2}10.11.19$ **M1**				
			$= 1045$ **A1**				
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(a)		$5\cosh x + 3\sinh x = 4$ $\Rightarrow 5\left(\frac{e^x + e^{-x}}{2}\right) + 3\left(\frac{e^x - e^{-x}}{2}\right) = 4$	M1	3.1a	Use of exponentials	Alternatively make cosh the subject, square and use Pythagoras to give quadratic in cosh Alternatively use compound angle formula
			$\Rightarrow 4e^x + e^{-x} = 4$ $\Rightarrow 4e^{2x} - 4e^x + 1 = 0 \left(\Rightarrow \left(2e^x - 1\right)^2 = 0\right)$	M1	3.1a	Multiply by e^x	
			$\Rightarrow e^x = \frac{1}{2}$	A1	1.1		
			$\Rightarrow x = -\ln 2$ **oe**	A1	1.1		
			Alternatively: $5\cosh x + 3\sinh x \equiv R\cosh(x+\alpha)$ **M1** where $R = \sqrt{25-9} = 4$, $\tanh\alpha = \frac{3}{5} \Rightarrow \alpha = \tanh^{-1}\frac{3}{5} = \frac{1}{2}\ln\left(\frac{1+\frac{3}{5}}{1-\frac{3}{5}}\right) = \frac{1}{2}\ln 4 = \ln 2$ **M1 A1** $\Rightarrow 4\cosh(x+\alpha) = 4 \Rightarrow \cosh(x+\alpha) = 0$ $\Rightarrow x = -\alpha = -\ln 2$ **A1**	[4]			

Question			Answer	Marks	AO	Guidance	
5	(b)		**DR** $\int_{-1}^{1}(5\cosh x+3\sinh x)\,dx=[5\sinh x+3\cosh x]_{-1}^{1}$ $=\left(5\frac{e^1-e^{-1}}{2}+3\frac{e^1+e^{-1}}{2}\right)-\left(5\frac{e^{-1}-e^1}{2}+3\frac{e^{-1}+e^1}{2}\right)$ $=(4e^1-e^{-1})-(4e^{-1}-e^1)$ $=5e-\frac{5}{e}$ Alternatively: $5\cosh x+3\sinh x$ $=5\left(\frac{e^x+e^{-x}}{2}\right)+3\left(\frac{e^x-e^{-x}}{2}\right)=\frac{1}{2}(8e^x+2e^{-x})$ **M1** $\Rightarrow\int_{-1}^{1}\frac{1}{2}(8e^x+2e^{-x})\,dx=[4e^x-e^{-x}]_{-1}^{1}$ **M1** $=(4e-e^{-1})-(4e^{-1}-e)=5e-\frac{5}{e}$ **A1**	**M1** **M1** **A1** **[3]**	1.1 1.1 2.1	Attempt at integral (i.e. one function changed) Convert sinh*x* and cosh*x* to exponential form in *their* integrated function and use limits correctly	Alternatively: M1 convert (including possibly using result from (a)) M1 integrate and use limits correctly

Question			Answer	Marks	AO	Guidance	
6	(a)		$=\frac{1}{1+\left(\sqrt{2x}\right)^2}\times\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{2x}\right)$ **oe** $=\frac{1}{1+2x}\times\frac{\sqrt{2}}{2\sqrt{x}}=\frac{1}{1+2x}\times\frac{1}{\sqrt{2x}}$	M1 A1 [2]	1.1a 1.1	Attempt to differentiate using chain rule	i.e. product of 2 terms
			Alternatively: $\tan y=\sqrt{2x}$ $\Rightarrow \sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{2}}{2\sqrt{x}}\Rightarrow\left(1+\tan^2 x\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{2}}{2\sqrt{x}}$ **M1** $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+2x}\times\frac{\sqrt{2}}{2\sqrt{x}}$ **A1**	[4]		Make a substitution	

Question			Answer	Marks	AO	Guidance	
6	(b)		$\int_{1/6}^{1/2} \frac{1}{(1+2x)\sqrt{x}}\,dx = \sqrt{2}\int_{1/6}^{1/2} \frac{1}{(1+2x)\sqrt{2x}}\,dx$	M1 A1	3.1a 1.1	Get into form of (a). Ignore limits Correct form	
			$= \sqrt{2}\left[\tan^{-1}\sqrt{2x}\right]_{1/6}^{1/2} = \sqrt{2}\left(\tan^{-1}1 - \tan^{-1}\frac{1}{\sqrt{3}}\right)$	M1	1.1	Use (a) and correct limits in correct order.	
			$= \sqrt{2}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{12}\pi$ So $k = \frac{\sqrt{2}}{12}$	A1	1.1	**oe**	
			Alternatively: Let $u = \sqrt{x}$ **M1** $du = \frac{1}{2\sqrt{x}}dx \Rightarrow dx = 2\sqrt{x}du = 2udu$ $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)}dx = \int_{x=\frac{1}{6}}^{x=\frac{1}{2}} \frac{u}{(u^2+2u^4)}2udu = 2\int_{x=\frac{1}{6}}^{x=\frac{1}{2}} \frac{1}{(1+2u^2)}du$ **A1** $= \sqrt{2}\left[\tan^{-1}u\sqrt{2}\right]_{x=\frac{1}{6}}^{x=\frac{1}{2}}$ **M1** $= \sqrt{2}\left[\tan^{-1}\sqrt{2x}\right]_{x=\frac{1}{6}}^{x=\frac{1}{2}} = \sqrt{2}\left(\tan^{-1}1 - \tan^{-1}\frac{1}{\sqrt{3}}\right) = \sqrt{2}\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ $= \frac{\pi\sqrt{2}}{12}$ **A1**	[4]		Make a substitution Get into correct form Use standard result with correct limits in correct order	

Question			Answer	Marks	AO	Guidance	
7	(a)		$\cosh x = \dfrac{e^x + e^{-x}}{2} = \dfrac{e^{2x}+1}{2e^x}$	**M1**	1.1	Use of cosh$x$ in exponentials	
			$\Rightarrow \operatorname{sech} x = \dfrac{2e^x}{e^{2x}+1}$ **AG**	**A1**	2.1		
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(b)		$u = e^x \Rightarrow du = e^x dx$ $\Rightarrow dx = \frac{du}{u}$	M1	3.1a	Substitute and use (a)	
			$\Rightarrow \int \text{sech}x\, dx = \int \left(\frac{2e^x}{e^{2x}+1}\right) dx$	A1	1.1	any form entirely in terms of u	Allow absence of du
			$= \int \frac{2u}{u^2+1} \cdot \frac{du}{u}$	M1	3.1a	Use standard form for integral and substitute back	
			$= 2\tan^{-1}(u) + c = 2\tan^{-1}(e^x) + c$	A1	1.1	Must include c	
			Alternatively: $u = \sinh x \Rightarrow du = \cosh x dx$ **M1** $\Rightarrow \int \text{sech}\, x dx = \int \frac{1}{\cosh x} \cdot \frac{du}{\cosh x} = \int \frac{du}{\cosh^2 x}$ $= \int \frac{du}{1+\sinh^2 x} = = \int \frac{du}{1+u^2}$ **A1** $= \tan^{-1} u + c$ **M1** $= \tan^{-1}(\sinh x) + c$ **A1**				
			Alternatively: $\int \text{sech}\, x dx = \int \frac{2e^x}{e^{2x}+1} dx$ Let $e^x = \tan u \Rightarrow e^x dx = \sec^2 u du \Rightarrow dx = \frac{\sec^2 u}{\tan u} du$ **M1** $\Rightarrow \int \text{sech}\, x dx = \int \frac{2\tan u}{\tan^2 u + 1} \cdot \frac{\sec^2 u}{\tan u} du = 2\int du$ **A1** $= 2u + c$ **M1** $= 2\tan^{-1}(e^x) + c$ **A1**	[4]		Substitute In correct form Integrate and substitute back Must include c	

Question			Answer	Marks	AO	Guidance	
8			AB has direction $\begin{pmatrix}4\\2\\1\end{pmatrix}$	B1	1.1	Direction	Alternative methods possible
			and any point on it is $(9+4\lambda,\ 6+2\lambda,\ 1+\lambda)$	M1	3.1a	Attempt to find point on line	
			If this point lies on plane then $4(9+4\lambda)+2(6+2\lambda)+(1+\lambda)=7$ $\Rightarrow 49+21\lambda=7 \Rightarrow 21\lambda=-42 \Rightarrow \lambda=-2$	M1	3.1a	Attempt to find λ	
			So B is where $\lambda=-4$	M1 A1	1.1a 1.1	Double λ λ **soi**	
			$\Rightarrow B$ has coordinates $(-7,-2,-3)$	A1	3.2a		Must be coordinates
			Alternatively: Distance between point and plane $=\frac{42}{\sqrt{21}}=2\sqrt{21}$ Distance between point and reflected point $=4\sqrt{21}$ **M1** Reflected point is $(x,y,z)\Rightarrow(x-9)^2+(y-6)^2+(z-1)^2$ Any point on normal line is $(9+4\lambda,6+2\lambda,1+\lambda)$ **B1** $\Rightarrow 16\lambda^2+4\lambda^2+\lambda^2=336 \Rightarrow 21\lambda^2=336 \Rightarrow \lambda^2=16$ **M1** $\Rightarrow \lambda=\pm4$ **A1** $(25,14,5)$ is the same side **A1** $\Rightarrow(-7,-2,-3)$	[6]			

Question			Answer	Marks	AO	Guidance	
9	(a)		**DR** $\omega = \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$ $\Rightarrow \omega^5 = \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^5 = \cos 2\pi + \mathrm{i}\sin 2\pi = 1 + 0\mathrm{i} = 1$	**M1** **A1** **[2]**	2.1 1.1	Finding ω^5 **AG**	Use of exponentials is satisfactory Could be argued backwards
	(b)		$\omega^2, \omega^3, \omega^4, 1$	**B1** **[1]**	1.1	Alternative: Roots are $\cos\frac{2k\pi}{5} + \mathrm{i}\sin\frac{2k\pi}{5}$ for $k = 2, 3, 4$ and 1(or $k = 5$)	Exponentials satisfactory
	(c)		**DR** $\omega^5 - 1 = 0$ $\Rightarrow (\omega - 1)(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$ $\Rightarrow \omega^4 + \omega^3 + \omega^2 + \omega = -1$ Alternatively: $1 + \omega + \omega^2 + \omega^3 + \omega^4 = \frac{1-\omega^5}{1-\omega} = \frac{0}{1-\omega}$ **M1** or $\omega + \omega^2 + \omega^3 + \omega^4 = \omega\left(\frac{1-\omega^4}{1-\omega}\right) = \left(\frac{\omega-\omega^5}{1-\omega}\right) = \left(\frac{\omega-1}{1-\omega}\right) = -1$ **A1** Alternatively: sum of roots = $= -\frac{b}{a}$ where $b = 0$ M1 – needs explanation – i.e. coefficient of z^4 term = 0	**M1** **A1** **[2]**	1.1a 2.1	Use equation and ω **AG**	

	(d)		**AG** $\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega+\frac{1}{\omega}\right)-1=\omega^2+2+\frac{1}{\omega^2}+\omega+\frac{1}{\omega}-1$ $=\frac{1}{\omega^2}\left(\omega^4+\omega^2+1+\omega^3+\omega\right)=0$ Since $\frac{1}{\omega^2}\neq 0,\ \omega^4+\omega^2+1+\omega^3+\omega=0$ or from part (c)	**M1** **A1** **A1**	2.1 1.1 2.2a	Multiply out	
			Alternatively: $\omega^4+\omega^3+\omega^2+\omega+1=0$ $\Rightarrow \omega^2\left(\omega^2+\omega+1+\frac{1}{\omega}+\frac{1}{\omega^2}\right)=0$ **M1** $\Rightarrow \omega^2\left(\left(\omega^2+2+\frac{1}{\omega^2}\right)+\left(\omega+\frac{1}{\omega}\right)+1-2\right)=0$ **A1** $\Rightarrow \omega^2\left(\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega+\frac{1}{\omega}\right)-1\right)=0$ Since $\omega^2\neq 0,\ \left(\omega+\frac{1}{\omega}\right)^2+\left(\omega+\frac{1}{\omega}\right)-1=0$ **A1**	**[3]**		For extraction of ω^2 For dealing with the 2	

	(e)		$\frac{1}{\omega} = \cos\frac{2\pi}{5} - \mathrm{i}\sin\frac{2\pi}{5}$ $\Rightarrow \left(\omega + \frac{1}{\omega}\right) = 2\cos\frac{2\pi}{5}$ From (iii) solving quadratic: $\left(\omega + \frac{1}{\omega}\right) = \frac{-1 \pm \sqrt{5}}{2}$ $\Rightarrow 2\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{2} \Rightarrow \cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$ $= -\frac{1}{4} + \frac{\sqrt{5}}{4}$ or $-\frac{1}{4} + \frac{1}{4}\sqrt{5}$ or $-0.25 + 0.25\sqrt{5}$	**B1** **B1** **M1** **A1**	3.1a 3.1a 2.2a 2.3	$\omega + \frac{1}{\omega}$ may be seen in (d) BC Equating For taking the valid value and presenting in correct form **oe**	No other forms acceptable
				[4]			

Question			Answer	Marks	AO	Guidance	
10	(a)		$\det\mathbf{A} = a^2 - 10a + 16$	M1 A1 [2]	1.1a 1.1	Attempt to work out the determinant	
	(b)		$a^2 - 10a + 16 = 0 \Rightarrow (a-2)(a-8) = 0 \Rightarrow a = 2,\ 8$	M1 A1 [2]	1.1a 1.1	Solving *their* quadratic soi	
	(c)		For both values there is no unique solution as det**A** = 0 For $a = 2$, equations are: $p_1 : 2x + 2y = 6$ $p_2 : 2y + 2z = 8$ $p_3 : 4x + 5y + z = 16$ $2p_1 + \frac{1}{2}p_2 = p_3$ So there is an infinite set of solutions. For $a = 8$, equations are: $p_1 : 8x + 2y = 6$ $p_2 : 8y + 2z = 8$ $p_3 : 4x + 5y + z = 16$ $\frac{1}{2}p_1 + \frac{1}{2}p_2 \neq p_3$ as it gives $4x + 5y + z = 7$ and $16 \neq 7$ so no solution	B1 M1 A1 A1 M1 A1 A1 [7]	2.4 2.1 1.1 2.2a 2.1 1.1 2.2a	**Soi** by correct answers Substitute one of *their* values and solve Substitute the other one of *their* values and solve	"correct answers" means solns are either infinite or non-existent.

Question			Answer	Marks	AO	Guidance	
11	(a)		$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=10\sin t$	M1	1.1a	For obtaining the AE	
			A.E. $n^2+2n+5=0 \Rightarrow n=-1\pm 2i$ $\Rightarrow x=e^{-t}\left(A\cos 2t+B\sin 2t\right)$	A1	1.1	Solving AE and interpreting	Or equivalent forms (i.e. exponential form)
			P.I. $x=a\sin t+b\cos t$ $\Rightarrow \frac{dx}{dt}=a\cos t-b\sin t$ $\Rightarrow \frac{d^2x}{dt^2}=-a\sin t-b\cos t$	M1	3.1a	Correct form of PI and differentiate twice	
			$\Rightarrow -a\sin t-b\cos t+2\left(a\cos t-b\sin t\right)$ $+5\left(a\sin t+b\cos t\right)=10\sin t$ $\Rightarrow -b+2a+5b=0,\ -a-2b+5a=10$	M1	1.1	**Dep on previous M** Substitute their PI with two constants into DE	
			i.e. $a=-2b,\ 2a-b=5 \Rightarrow b=-1,\ a=2$	A1	1.1	Values of a and b	
			$\Rightarrow$ G.S. $x=e^{-t}\left(A\cos 2t+B\sin 2t\right)+2\sin t-\cos t$	A1	1.1	Correct GS	
			When $t=0$ we have $x=2$ and $\frac{dx}{dt}=0$	B1	3.3	For substituting into 2 eqns	
			$\Rightarrow A-1=2 \Rightarrow A=3$	B1	3.3	For A	
			$\frac{dx}{dt}=-e^{-t}\left(A\cos 2t+B\sin 2t\right)$ $+e^{-t}\left(-2A\sin 2t+2B\cos 2t\right)+2\cos t+\sin t$ $t=0 \Rightarrow -A+2B+2=0 \Rightarrow B=\frac{1}{2}$	M1	3.4	**Dep on previous M** For diffn *their* GS. Their value for A could be substituted	
			$\Rightarrow x=e^{-t}\left(3\cos 2t+\frac{1}{2}\sin 2t\right)+2\sin t-\cos t$	A1	3.3	For B	
				A1 **[11]**	3.4		

	(b)		$x = e^{-t}\left(A\cos 2t + B\sin 2t\right) + 2\sin t - \cos t$ As $t \to \infty$, $e^{-t} \to 0$ $\Rightarrow x \approx 2\sin t - \cos t$	**M1** **A1** **[2]**	3.4 3.4	Consider behaviour of e^{-t} Accept = **ft** *their* equation from (a)	Accept this final line for 2 marks

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